Adding and Subtracting

When adding or subtracting values, add the absolute uncertainties to give the absolute uncertainty in the result.

If $y = a \pm b$ then: $\Delta y = \Delta a + \Delta b$

$y = a + b$ or $y = a - b$

for adding or subtracting

You always add the absolute uncertainties

Example:

$(9.7 \pm 0.5)\,m + (4.3 \pm 0.2)\,m = (14.0 \pm 0.7)\,m$

$(9.7 \pm 0.5)\,m - (4.3 \pm 0.2)\,m = (5.4 \pm 0.7)\,m$

Example: Determine the perimeter of a square of side (2.4 ± 0.5) cm

$$\begin{array}{rl} & 2.4 \pm 0.5 \text{ cm} \\ & 2.4 \pm 0.5 \text{ cm} \\ & 2.4 \pm 0.5 \text{ cm} \\ + & 2.4 \pm 0.5 \text{ cm} \\ \hline & 9.6 \pm 2.0 \text{ cm} \end{array}$$

(10 ± 2) cm

or

$4\,(2.4 \pm 0.5)$ cm

(9.6 ± 2.0) cm

(10 ± 2) cm

Multiplying & Dividing

When multiplying & dividing, add the relative uncertainties to get the relative uncertainty of the result.

If $y = \frac{ab}{c}$ then $\frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c}$

relative uncertainty

Example:

$(9.7 \pm 0.5)m \times (4.3 \pm 0.2)m = 41.7 m^2$
$\pm$??

add the relative uncertainties $\frac{0.5}{9.7} + \frac{0.2}{4.3} = 0.052 + 0.047 = 0.099$

relative uncertainty of $41.7 m^2$

absolute uncertainty for $41.7 m^2$ $0.099(41.7 m^2) = 4.1283 m^2$

relative unct.

absolute uncertainty

Final answer: $(41.7 \pm 4) m^2$

same place value

$(42 \pm 4) m^2$

Example: Determine the answer with its absolute uncertainty:

$$\frac{(9.7 \pm 0.5)m}{(4.3 \pm 0.2)m} = 2.2558 \pm ??$$

Add relative uncertainties: $0.052 + 0.047 = 0.099$

relative uncertainty for 2.2558

absolute uncertainty in final answer: $0.099(2.2558) = 0.2233242$

can only have 1 sd

Final answer:

(2.2558 ± 0.2)

(2.3 ± 0.2)

Powers (and roots)

When raising a value to the power of n, multiply the relative uncertainty by n to give the relative uncertainty of the result.

Example: $\left[(9.7 \pm 0.5)\text{m}\right]^{3} = 912.673\text{m}^3 \pm ??$

relative uncertainty for 9.7: $\frac{0.5}{9.7} = 0.052$

relative uncertainty for $(9.7)^3$: $3(0.052) = 0.156$ ← relative uncertainty for the final answer. (912.673)

absolute uncertainty in $(9.7)^3$: $0.156(912.673) = 141.135$

$(912.673 \pm 141.135)\text{m}^3$

$(900 \pm 100)\text{m}^3$

$(9 \pm 1) \times 10^2\ \text{m}^3$

Example The radius of a sphere is measured to be (8.5 ± 0.2)cm. Determine its volume with its absolute uncertainty.

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi (8.5)^3$

$\boxed{V = 2572.4\ \text{cm}^3}$

relative uncertainty 8.5 cm: $\frac{0.2}{8.5} = 0.0235$

relative uncertainty $(8.5\text{cm})^3$: $3(0.0235) = 0.07059$

absolute uncertainty for $(8.5\text{cm})^3$ $= 0.07059(8.5\text{cm})^3 = 43.351\ldots$

$V = \frac{4}{3}\pi(614.125 \pm 43.351)\ \text{cm}^3$

$V = (2572.4 \pm 181.59)\ \text{cm}^3$

$V = (2.6 \pm 0.2) \times 10^3\ \text{cm}^3$

EASIER to find the absolute uncertainty for the final answer by:

$0.07059(2572.4)$

$= 181.59$ (the same)

Example:

The surface area of a square swimming pool is found to be $12 m^2$ with an absolute uncertainty of $2 m^2$. Determine the length of each side of the pool with its absolute error.

$$\text{Area} = (\text{length})^2$$

$$\text{length} = \sqrt{\text{Area}}$$

$$\text{length} = \text{Area}^{1/2}$$

$$\text{length} = \sqrt{12 m^2}$$

$$\text{length} = 3.464\ldots$$

relative uncertainty area : $\frac{2}{12} = 0.17$

relative uncertainty side length : $\frac{1}{2}(0.17) = 0.0833$

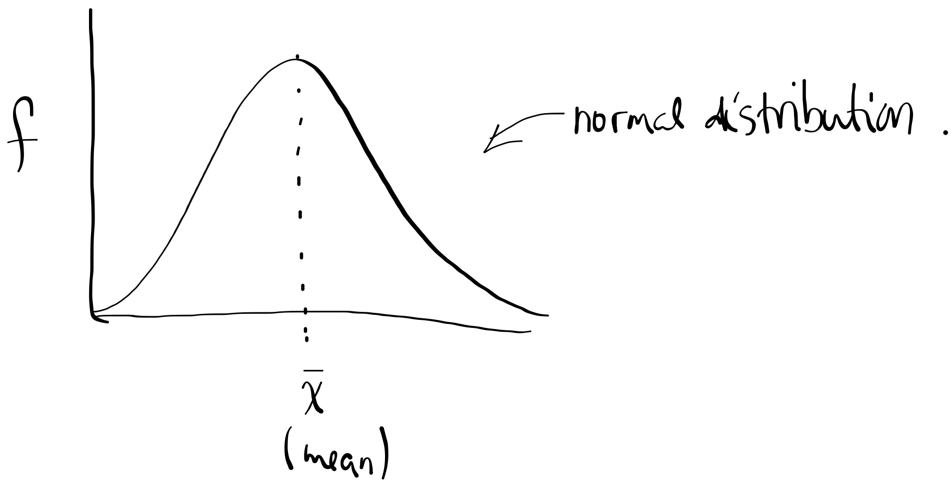
absolute uncertainty side : $0.0833(3.464) = 0.2847\ldots$

final answer

$$(3.464 \pm 0.2847)\, m$$

$$(3.5 \pm 0.3)\, m$$

If you take repeated measurements of the same thing, the measurements will follow a normal distribution.



If we have a large sample size, the uncertainty is basically the standard deviation.

Usually, in the lab, we might only take a sample of 5. Instead of using the standard deviation as a measure of the uncertainty, we can use $\frac{1}{2}$ of the range.

$$\bar{x} \pm \frac{(x_{max} - x_{min})}{2}$$

$$\text{mean} \pm \frac{\text{range}}{2}$$